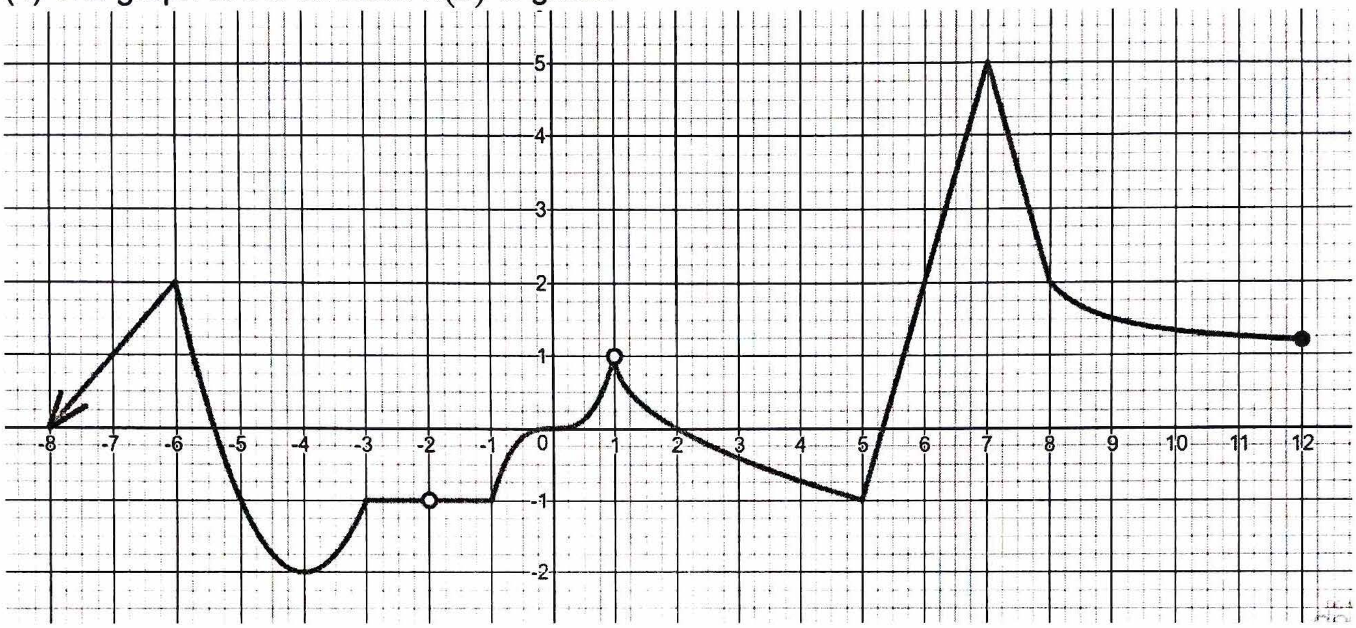
AP Calculus BC
Summer Assignment

Name ANSWER KEY **Date** ______

Show all work! Exact answers only unless the problem asks for an approximation.

Determine the domain & range and evaluate each expression for the function. Then determine the value(s) of x at the local & absolute minima & maxima of the function and find the interval(s) on which the function is increasing & decreasing.

(1) The graph of the function $h(x)$ is given.



(A) Domain $(-\infty,-2)\cup(-2,1)\cup(1,12]$ (B) Range $(-\infty,5]$

(C) $h(-4)=$ -2 (D) Find x so that $h(x)=2$ $x=$ $-6, 6, 8$

(E) Local minima $x=-4, 5, 12$ (F) Local maxima $x=-6, 7$

(G) Absolute minima ϕ (H) Absolute maxima $x=7$

(I) Interval(s) of decrease $[-6,-4]\cup(1,5]\cup[7,12]$ OR $(-6,-4)\cup(1,5)\cup(7,12)$

(J) Interval(s) of increase $(-\infty,-6]\cup[-4,-3]\cup[-1,1)\cup[5,7]$ OR $(-\infty,-6)\cup(-4,-3)\cup(-1,1)\cup(5,7)$

Determine algebraically whether each function is even, odd or neither.

(2) $f(x)=\dfrac{8}{x}-\dfrac{7}{x^3}+\dfrac{9}{\sqrt[3]{x}}$

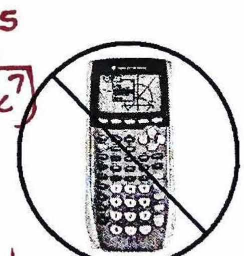
$$f(-x) = -\frac{8}{x}+\frac{7}{x^3}-\frac{9}{\sqrt[3]{x}}$$

ODD FUNCTION

(3) $g(x)=\dfrac{6x^5}{2x^3-4x^7}$

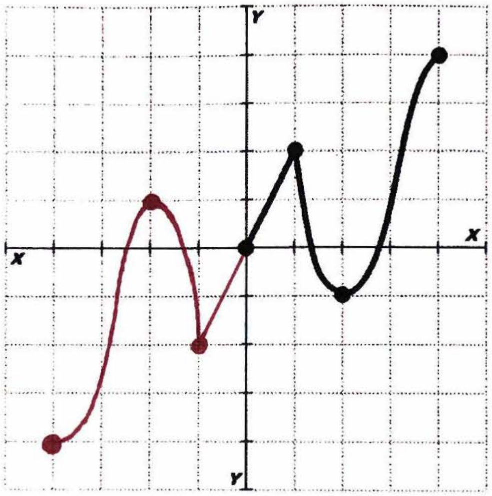
$$g(-x) = \frac{-6x^5}{-2x^3+4x^7} = \frac{-6x^5}{-(2x^3-4x^7)} = \frac{6x^5}{2x^3-4x^7}$$

EVEN FUNCTION

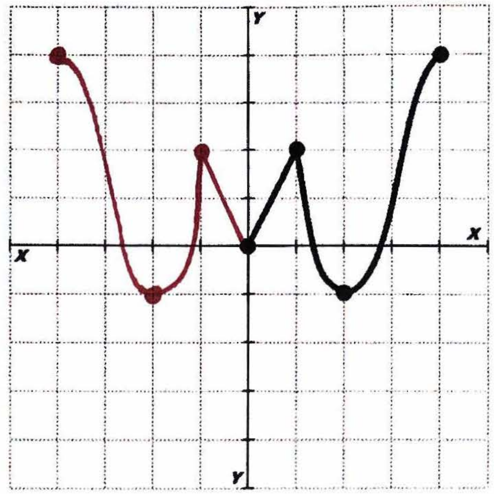


(4) A portion of the graph of a function $g(x)$ defined on $[-4,4]$ is shown.

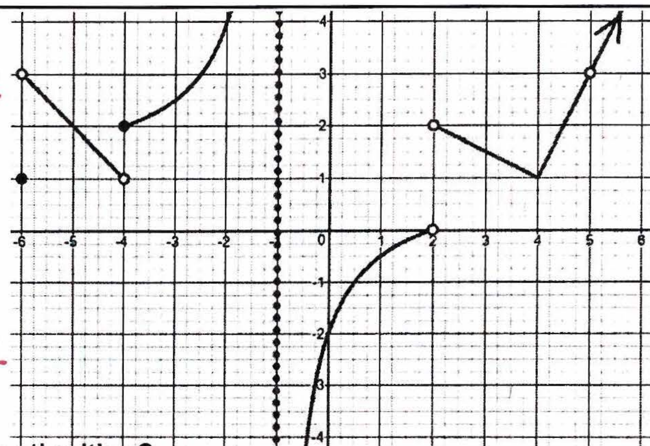
(A) Complete the graph if $g(x)$ is odd



(B) Complete the graph if $g(x)$ is even



(5) Find the limits given the graph of $f(x)$.



(A) $\lim_{x\to -4^-} f(x) =$ 1

(B) $\lim_{x\to 2} f(x) =$ DNE

(C) $\lim_{x\to -1^+} f(x) =$ $-\infty$

(D) $\lim_{x\to 5} f(x) =$ 3

(E) $\lim_{x\to 0^-} f(x) =$ -2

(F) $\lim_{x\to -6} f(x) =$ 3

(G) $\lim_{x\to \infty} f(x) =$ ∞

(H) $\lim_{x\to -1} f(x) =$ DNE

(I) At what values of x are there removable discontinuities?

$x = -6, 5$

(J) At what values of x are there nonremovable discontinuities?

$x = -4, -1, 2$

(6) Sketch a graph of a function $g(x)$ that satisfies all of the following conditions (use dashed lines to represent any asymptotes):

(A) $g(x)$ is increasing on $[2,5)$ and $[8,10]$

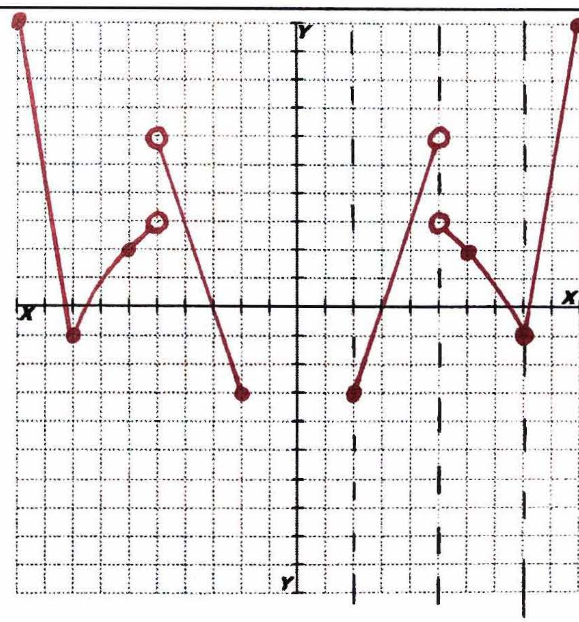
(B) $g(x)$ is decreasing on $(5,8]$

(C) $g(x)$ is an even function

(D) $g(x)$ has an absolute maximum at $x = 10$

(E) $g(x)$ has a local (not absolute) minimum at $x = 8$

(F) $g(6) = 2$



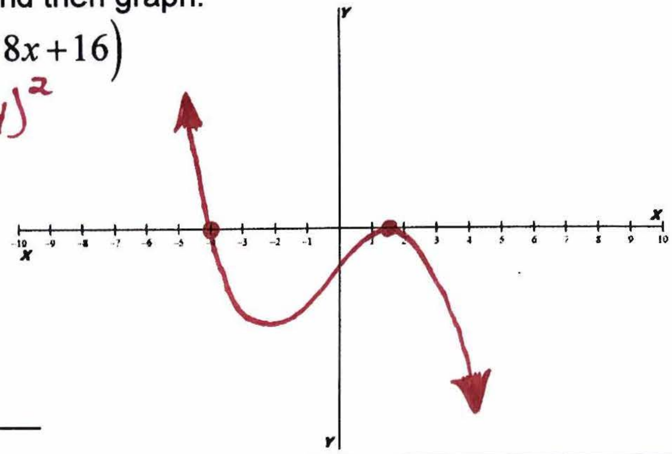
(7) For the polynomial function, list each real zero & its multiplicity, determine whether the graph crosses or touches the x-axis at each x-intercept, determine the end behavior model and describe the end behavior using limits and then graph.

$$g(x) = -2(8x^3-27)(2x^2+5x-12)(x^2+8x+16)$$

$= -2(2x-3)(4x^2+6x+9)(2x-3)(x+4)(x+4)^2$

$= -2(2x-3)^2(x+4)^3(4x^2+6x+9)$

NON REAL ZEROS

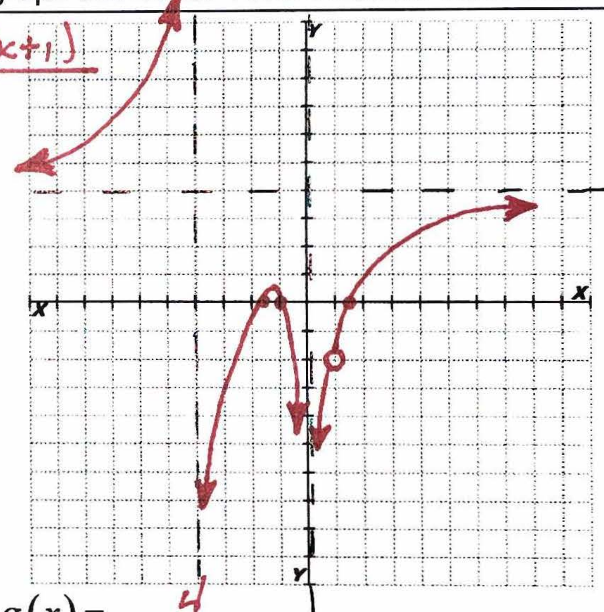


End behavior model $y = -x^7$

$\lim_{x\to\infty} g(x) = -\infty$ $\lim_{x\to -\infty} g(x) = \infty$

Identify the hole(s), vertical asymptote(s), horizontal asymptote, oblique asymptote, x-intercept(s) & y-intercept of each rational function. Then graph each rational function using **dashed lines for the asymptotes**. Do not forget to graph the holes. Then evaluate each limit.

(8) $g(x) = \dfrac{4x^4-13x^2+9}{x^4+3x^3-4x^2} = \dfrac{(2x-3)(2x+3)(x-1)(x+1)}{x^2(x+4)(x-1)}$



Hole(s) $(1,-2)$

Vertical asymptote(s) $x = 0, -4$

Horizontal asymptote $y = 4$

Oblique asymptote ϕ

X-intercept(s) $x = -3/2, -1, 3/2$

Y-intercept ϕ

$\lim_{x\to -\infty} g(x) = 4$ $\lim_{x\to\infty} g(x) = 4$

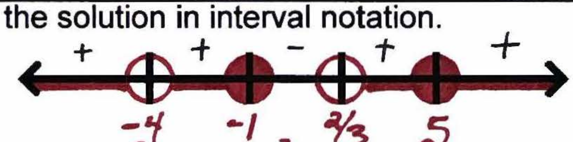
$\lim_{x\to 0^+} g(x) = -\infty$ $\lim_{x\to 0} g(x) = $ DNE

$\lim_{x\to 1^-} g(x) = -2$ $\lim_{x\to -4^-} g(x) = \infty$

$\lim_{x\to -4^+} g(x) = -\infty$ $\lim_{x\to 1} g(x) = -2$

(9) Solve the inequality, graph the solution set & write the solution in interval notation.

$$\frac{(x+1)^3(x^2-10x+25)}{(3x^2+10x-8)(x+4)} \geq 0$$



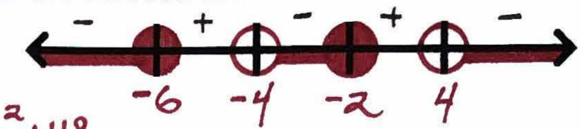
$\dfrac{(x+1)^3(x-5)^2}{(3x-2)(x+4)(x+4)} \geq 0$

$\dfrac{(x+1)^3(x-5)^2}{(3x-2)(x+4)^2} \geq 0$

Interval Notation $(-\infty,-4)\cup(-4,-1]\cup(\frac{2}{3},\infty)$

(10) Solve the inequality, graph the solution set & write the solution in interval notation.

$$\frac{12}{x^2-16}-\frac{24}{x-4}\le 3$$



$$\frac{12-24(x+4)-3(x^2-16)}{x^2-16}\le 0 \rightarrow \frac{12-24x-96-3x^2+48}{x^2-16}\le 0 \rightarrow \frac{-3(x+2)(x+6)}{(x-4)(x+4)}\le 0$$

Interval Notation $(-\infty,-6]\cup(-4,-2]\cup(4,\infty)$

(11) Determine the value of k that makes the function continuous.

$$g(x)=\begin{cases} kx^2-4x-3, & x\le -2 \\ \frac{3}{2}x+6, & x>-2 \end{cases}$$

$K(-2)^2-4(-2)-3=3$

$4K=-2$

$\frac{3}{2}(-2)+6=3$

$K=-\frac{1}{2}$

(12) Sketch a graph of a function $h(x)$ that satisfies all of the following conditions:

(A) $\lim\limits_{x\to-\infty} h(x)=-5$;

(B) $\lim\limits_{x\to\infty} h(x)=-\infty$;

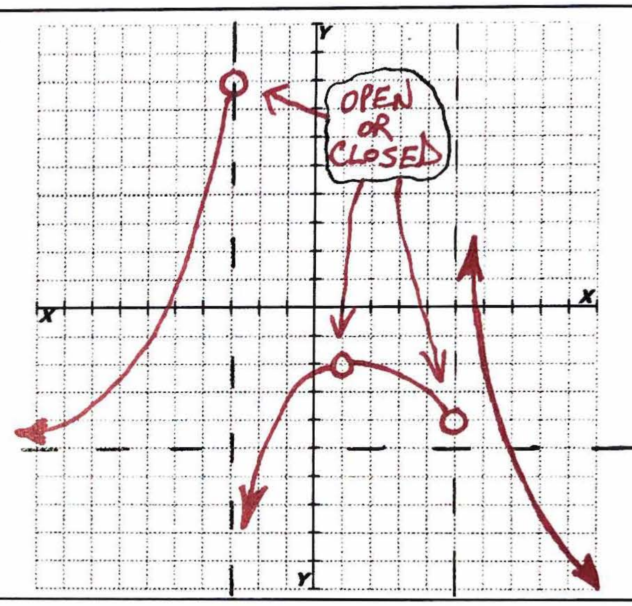
(C) $\lim\limits_{x\to-3^+} h(x)=-\infty$;

(D) $\lim\limits_{x\to-3^-} h(x)=8$;

(E) $\lim\limits_{x\to5^-} h(x)=-4$;

(F) $\lim\limits_{x\to5^+} h(x)=\infty$;

(G) $\lim\limits_{x\to1} h(x)=-2$



Evaluate each limit.

(13) $\lim\limits_{x\to0}\left(\frac{\sqrt{x+1}-1}{x}\right)=\frac{1}{2}$

$$\lim_{x\to0}\frac{x+1-1}{x(\sqrt{x+1}+1)}=\lim_{x\to0}\frac{1}{\sqrt{x+1}+1}$$

(14) $\lim\limits_{x\to0}\left(\frac{\frac{1}{x+2}-\frac{1}{2}}{x}\right)=-\frac{1}{4}$

$$\lim_{x\to0}\frac{2-x-2}{2x(x+2)}=\lim_{x\to0}\frac{-1}{2(x+2)}$$

Find a formula for $f^{-1}(x)$ & $g^{-1}(x)$ then determine the domain and range of their inverse.

(15) $f(x)=-\sqrt{x-1}-4$ D: $[1,\infty)$ R: $(-\infty,-4]$

$x=-\sqrt{y-1}-4$

$\sqrt{y-1}=-x-4$

$y-1=x^2+8x+16$

$f^{-1}(x)=x^2+8x+17$

DOMAIN $(-\infty,-4]$

RANGE $[1,\infty)$

(16) $g(x)=\frac{5-3x}{x+4}$ D: $x\neq-4$ R: $y\neq-3$

$x=\frac{5-3y}{y+4}$

$xy+4x=5-3y$

$y(x+3)=5-4x$

$g^{-1}(x)=\frac{5-4x}{x+3}$

DOMAIN $(-\infty,-3)\cup(-3,\infty)$

RANGE $(-\infty,-4)\cup(-4,\infty)$

Solve each equation.

(17) $\ln(6x) - \ln(x+5) + 3\ln 4 = 2$

$\ln\left(\frac{6x \cdot 64}{x+5}\right) = 2$

$\frac{384x}{x+5} = e^2$

$384x = e^2 x + 5e^2$

$x(384 - e^2) = 5e^2$

$x = \frac{5e^2}{384 - e^2}$

(18) $\pi^{2x+3} = e^{x-4}$

$2x\ln\pi + 3\ln\pi = x - 4$

$x(2\ln\pi - 1) = -4 - 3\ln\pi$

$x = \frac{-4-3\ln\pi}{2\ln\pi - 1}$ OR $\frac{4+3\ln\pi}{1-2\ln\pi}$

(19) Determine the logistic function $P(x) = \frac{c}{1 + a \bullet b^x}$ whose initial population is 4, carrying capacity is 36 and passing through (12, 24).

$(0,4)$

$4 = \frac{36}{1+a}$

$1+a = 9 \rightarrow a = 8$

$24 = \frac{36}{1+8b^{12}}$

$1 + 8b^{12} = 3/2$

$b^{12} = \frac{1}{16}$

$b = (2^{-4})^{\frac{1}{12}} = 2^{-1/3}$

$P(x) = \frac{36}{1 + 8 \cdot \sqrt[3]{\frac{1}{2}}^{\,x}}$

Find the exact value of each expression.

(20) $\cot\left(\frac{11\pi}{3}\right) =$ $-1/\sqrt{3}$

(21) $\sec\left(-\frac{7\pi}{6}\right) =$ $-2/\sqrt{3}$

(22) $\sin\left(\frac{15\pi}{2}\right) =$ -1

(23) $\csc\left(-\frac{13\pi}{4}\right) =$ $\sqrt{2}$

(24) $\tan\left(\frac{9\pi}{2}\right) =$ UND

(25) $\cos\left(-\frac{13\pi}{3}\right) =$ $1/2$

(26) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ $5\pi/6$

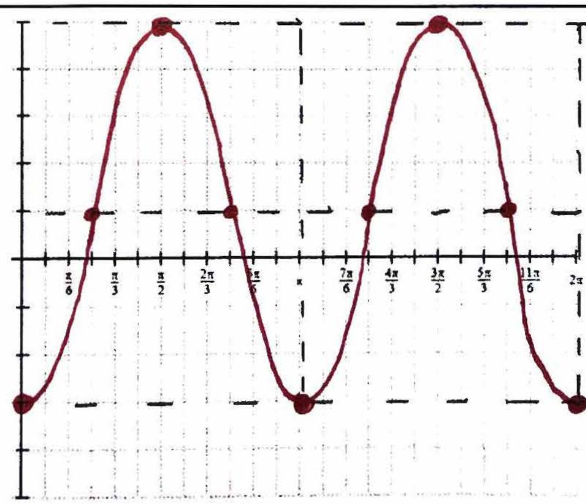
(27) $\tan^{-1}(0) =$ 0

(28) $\sin^{-1}\left(-\frac{1}{2}\right) =$ $-\pi/6$

(29) $\tan^{-1}\left(-\sqrt{3}\right) =$ $-\pi/3$

Graph each trigonometric function.

(30) $f(x) = -4\cos(2x) + 1$



Solve each equation on the interval $[0,2\pi)$.

(31) $2+5\csc(4\theta)=-1+2\csc(4\theta)$

$3\csc(4\theta)=-3$

$\csc(4\theta)=-1$

$\sin(4\theta)=-1$

$4\theta=\frac{3\pi}{2} \rightarrow \theta=\frac{3\pi}{8}$

$4\theta=\frac{7\pi}{2} \rightarrow \theta=\frac{7\pi}{8}$

$4\theta=\frac{11\pi}{2} \rightarrow \theta=\frac{11\pi}{8}$

$4\theta=\frac{15\pi}{2} \rightarrow \theta=\frac{15\pi}{8}$

(32) $\cot\theta+\cos\theta=-2\cot\theta \bullet \cos\theta+\cos\theta$

$\cot\theta+2\cot\theta\cos\theta=0$

$\cot\theta(1+2\cos\theta)=0$

$\cot\theta=0 \qquad \cos\theta=-\frac{1}{2}$

$\theta=\frac{\pi}{2}, \frac{3\pi}{2} \qquad \theta=\frac{2\pi}{3}, \frac{4\pi}{3}$

(33) $3=3\sin\theta+2\sin^2\theta+4$

$0=2\sin^2\theta+3\sin\theta+1$

$0=(2\sin\theta+1)(\sin\theta+1)$

$\sin\theta=-\frac{1}{2} \qquad \sin\theta=-1$

$\theta=\frac{7\pi}{6}, \frac{11\pi}{6} \qquad \theta=\frac{3\pi}{2}$

(34) $\sec\theta=1-\tan^2\theta$

$\sec\theta=1-(\sec^2\theta-1)$

$\sec^2\theta+\sec\theta-2=0$

$(\sec\theta+2)(\sec\theta-1)=0$

$\cos\theta=-\frac{1}{2}$

$\theta=\frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos\theta=1$

$\theta=0$

(35) Simplify the expression.

$\cos\theta \bullet \cot\theta \bullet \sin\theta \bullet \sec\theta =$ $\underline{\cos\theta}$

$\cos\theta \bullet \frac{\cos\theta}{\sin\theta} \bullet \sin\theta \bullet \frac{1}{\cos\theta}$

(36) Simplify the expression.

$\frac{\sin^2\theta+\tan^2\theta+\cos^2\theta}{\sec\theta}=$ $\underline{\sec\theta}$

$\frac{1+\tan^2\theta}{\sec\theta}=\frac{\sec^2\theta}{\sec\theta}$

(37) Simplify the expression.

$\frac{1+\tan\theta}{\sin\theta+\cos\theta}=$ $\underline{\sec\theta}$

$\frac{1+\frac{\sin\theta}{\cos\theta}}{\sin\theta+\cos\theta}=\frac{\cos\theta+\sin\theta}{\cos\theta(\sin\theta+\cos\theta)}$

(38) Simplify the expression.

$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=$ $\underline{2\csc\theta}$

$\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}=\frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}=\frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

(39) Simplify the expression.

$\frac{1-\tan^2\theta}{\tan^2\theta+1}+1=$ $\underline{2\cos^2\theta}$

$\frac{1-\tan^2\theta+\tan^2\theta+1}{\tan^2\theta+1}=\frac{2}{\sec^2\theta}$

Find all solutions to the equations in the interval $[0,2\pi)$.

(40) $\sin(2\theta)\bullet\cos\theta=\sin\theta$

$2\sin\theta\cos^2\theta-\sin\theta=0$

$\sin\theta\left(2\cos^2\theta-1\right)=0$

$\sin\theta=0 \qquad \cos^2\theta=1/2$

$\theta=0,\pi \qquad \cos\theta=\pm\sqrt{2}/2$

$\theta=\pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$

(41) $\cos(2\theta)=4\cos\theta-3$

$2\cos^2\theta-1=4\cos\theta-3$

$2\cos^2\theta-4\cos\theta+2=0$

$2(\cos^2\theta-2\cos\theta+1)=0$

$2(\cos\theta-1)^2=0$

$\cos\theta=1$

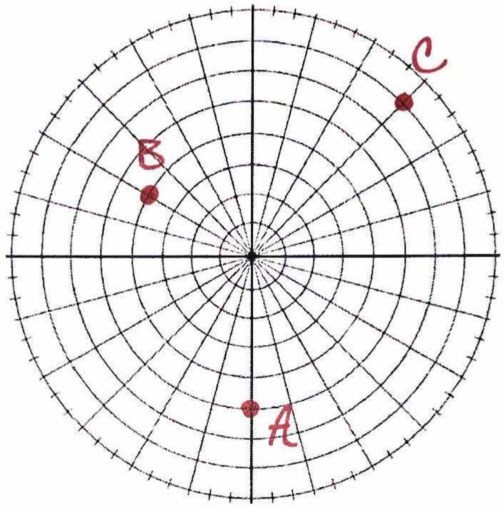
$\theta=0$

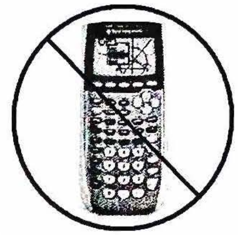
(42) Plot the points with the given polar coordinates.

(A) $\left(5,\dfrac{11\pi}{2}\right)$

(B) $\left(-4,\dfrac{23\pi}{6}\right)$

(C) $\left(7,-\dfrac{7\pi}{4}\right)$





(43) Find the rectangular coordinates of the points with the given polar coordinates.

(A) $\left(5,\dfrac{11\pi}{2}\right)$ $(0,-5)$

(B) $\left(-4,\dfrac{23\pi}{6}\right)$ $(-2\sqrt{3},2)$

(C) $\left(7,-\dfrac{7\pi}{4}\right)$ $\left(\dfrac{7}{\sqrt{2}},\dfrac{7}{\sqrt{2}}\right)$

(44) Find the polar coordinates (radians) of the points with the given rectangular coordinates.

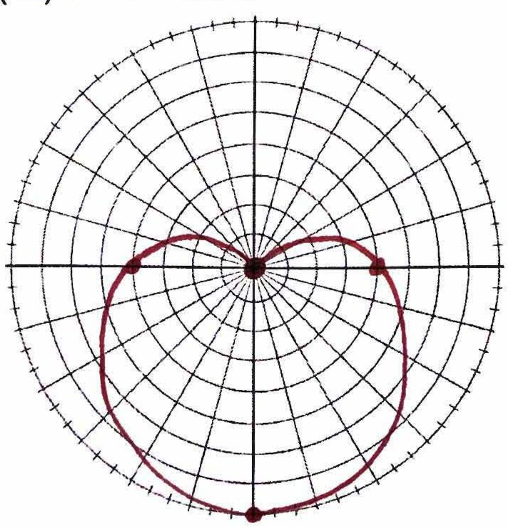
(A) $(-5,5)$ $(5\sqrt{2},3\pi/4)$

(B) $(-4,-4\sqrt{3})$ $(8,4\pi/3)$

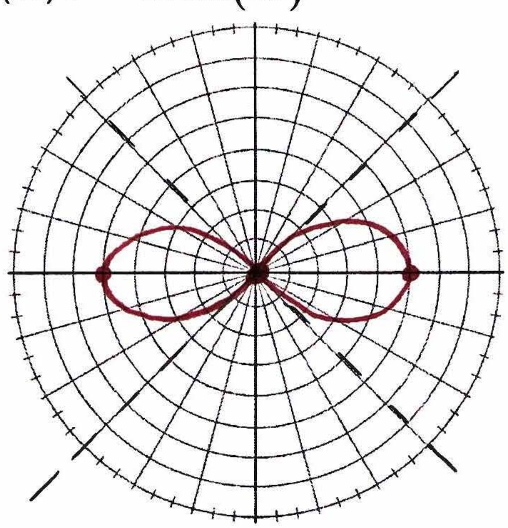
(C) $(-8,0)$ $(8,\pi)$

Graph the polar equation and classify the curve.

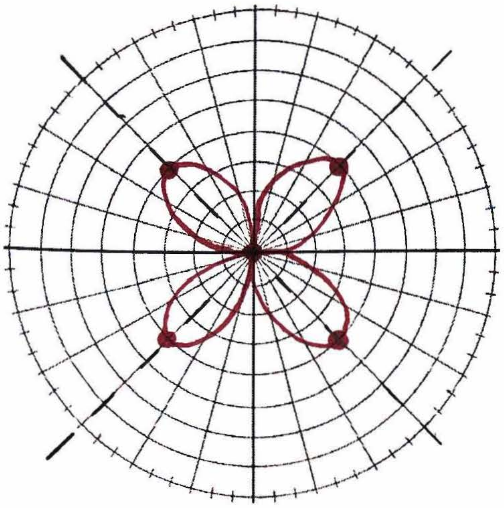
(45) $r=4-4\sin\theta$



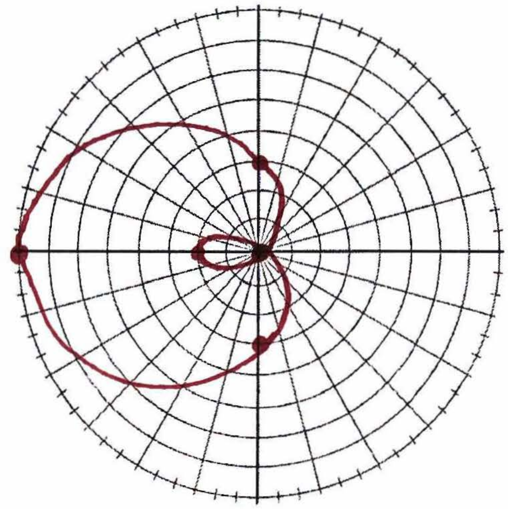
(46) $r^2=25\cos(2\theta)$



(47) $r = 4\sin(2\theta)$



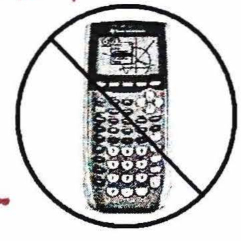
(48) $r = 3 - 5\cos\theta$



(49) Find the partial fraction decomposition of the rational expression.

$$\frac{x^2-19x+17}{(x-4)(2x^2+3x-1)} = \frac{A}{x-4} + \frac{Bx+C}{2x^2+3x-1} = \frac{-1}{x-4} + \frac{3x-4}{2x^2+3x-1}$$

$2Ax^2+3Ax-A+Bx^2-4Bx+Cx-4C = 1x^2-19x+17$



$2A+B = 1 \longrightarrow 32A+16B=16$

$3A-4B+C=-19 \longrightarrow 12A-16B+4C=-76$

$-A \quad -4C=17 \longrightarrow -A \quad -4C=17$

$11A-16B=-59$

$11A-16B=-59$

$43A=-43$

$A=-1,\ B=3,\ C=-4$

(50) Rewrite the improper rational expression as the sum of a polynomial and a proper rational expression, then find the partial fraction decomposition of the proper rational expression. Finally, express the improper rational expression as the sum of a polynomial and the partial fraction decomposition.

$$\frac{x^3-5x^2+21}{x^2-x-2} = x-4 + \frac{-2x+13}{(x-2)(x+1)} = x-4 + \frac{3}{x-2} + \frac{-5}{x+1}$$

$$\begin{array}{r} x-4 \\ x^2-x-2\,\big)\,\overline{x^3-5x^2+0x+21} \\ \underline{x^3-x^2-2x} \\ -4x^2+2x+21 \\ \underline{-4x^2+4x+8} \\ -2x+13 \end{array}$$

$$\frac{-2x+13}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$-2x+13 = Ax+A+Bx-2B$

$A+B=-2$

$A-2B=13$

$3B=-15$

$B=-5 \quad A=3$

(51) Find the average rate of change of each function over the given interval.

$f(x) = -x^2+2x;\ [-1,4]$

$(-1,-3)$

$(4,-8)$

SLOPE OF SECANT LINE $= \dfrac{-3 - -8}{-1-4} = \dfrac{5}{-5} = -1$

Average rate of change = -1

Use the definition of the derivative to find the derivative of the each function. Then find the instantaneous rate of change of each function at the given value of x.

(52) $g(x)=\dfrac{-4}{x+2}$

$$\lim_{h\to 0}\frac{\frac{-4}{x+h+2}-\frac{-4}{x+2}}{h}=$$

$$\lim_{h\to 0}\frac{\cancel{-4x}\cancel{-8}+\cancel{4x}+4h\cancel{+8}}{h(x+2)(x+h+2)}$$

Derivative $g'(x)=$ $\dfrac{4}{(x+2)^2}$

Instantaneous rate of change at $x=-3$ $\quad slope=$ 4

(53) $h(x)=\sqrt{2x+7}$

$$\lim_{h\to 0}\frac{\sqrt{2x+2h+7}-\sqrt{2x+7}}{h}=\lim_{h\to 0}\frac{\cancel{2x}+2h\cancel{+7}-\cancel{2x}\cancel{-7}}{h\left(\sqrt{2x+2h+7}+\sqrt{2x+7}\right)}$$

$$=\frac{2}{2\sqrt{2x+7}}$$

Derivative $h'(x)=$ $\dfrac{1}{\sqrt{2x+7}}$

Instantaneous rate of change at $x=1$ $\quad slope=$ $1/3$

(54) Write the equation in standard form and classify the conic section. Identify the important characteristics of the graph.

$$x^2+16y^2+10x-64y+25=0$$

$$(x^2+10x+25)+16(y^2-4y+4)=-25+25+64$$

$$(x+5)^2+16(y-2)^2=64 \longrightarrow \frac{(x+5)^2}{64}+\frac{(y-2)^2}{4}=1$$

ELLIPSE
CENTER $(-5,2)$
MAJOR AXIS $=16$
MINOR AXIS $=4$
VERTICES $(3,2)$ $(-13,2)$
$(-5,4)$ $(-5,0)$

(55) Write the equation in standard form and classify the conic section. Identify the important characteristics of the graph.

$$x^2+y^2+4x-6y-87=0$$

$$(x^2+4x+4)+(y^2-6y+9)=87+4+9$$

$$(x+2)^2+(y-3)^2=100$$

CIRCLE
CENTER $(-2,3)$
RADIUS $=10$

(56) Determine the values a and b that make $f(x)$ differentiable at $x=-1$. Use the Definition of the Derivative or the <u>Alternative Form</u>.

$$f(x)=\begin{cases}\dfrac{2}{x-1}, & x>-1\\ ax^2+b, & x\le -1\end{cases}$$

$$\lim_{x\to -1^+}\frac{\frac{2}{x-1}+1}{x+1}=\lim_{x\to -1^-}\frac{ax^2+\cancel{b}-a-\cancel{b}}{x+1}$$

$$\lim_{x\to -1^+}\frac{\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}=\lim_{x\to -1^-}\frac{a(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$

NEEDS TO BE CONTINUOUS

$a+b=-1$

$-1/2=-2a$

$1/4=a \qquad b=-5/4$

AP Calculus BC

Summer Assignment

Name__________________________ **Date**__________________________

Show all work! Exact answers only unless the problem asks for an approximation.

(57) Find the value(s) of x at the local maxima & minima. Then identify the interval(s) on which the function is increasing or decreasing.

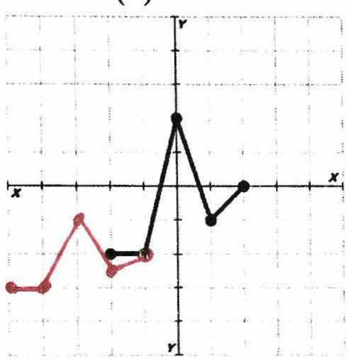
$$g(x) = -\frac{x^2 - 2x + 1}{x + 1}$$

Maxima $x = 1$ Minima $x = -3$

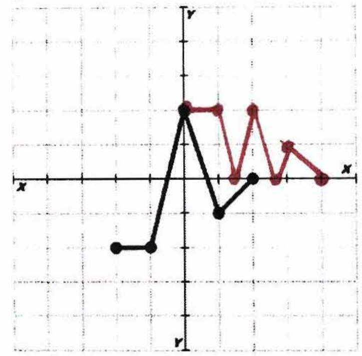
Increasing $(-3,-1) \cup (-1,1)$ Decreasing $(-\infty,-3) \cup (1,\infty)$

(58) The graph of a function $h(x)$ is shown below. Draw the graph of the following:

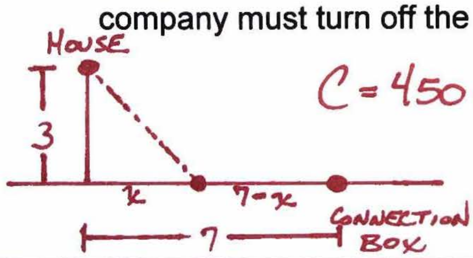
(A) $\frac{1}{2} \bullet h(x+3) - 2$



(B) $|h(x-2)|$



(59) MetroMedia Cable is asked to provide service to a customer whose house is located 3 miles from the road along which the cable is buried. The nearest connection box for the cable is located 7 miles down the road. If the installation cost is \$300 per mile along the road and \$450 per mile off the road, build a model that expresses the total cost of installation as a function of the distance from the connection box to the point where the cable installation turns off the road. Find the distance from the connection box that the company must turn off the road to minimize the cost. What is the minimum cost?

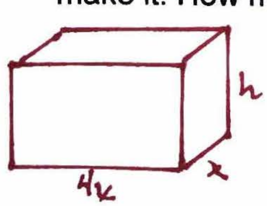


$C = 450\sqrt{x^2+9} + 300(7-x)$ MINIMUM $(\underset{x}{2.683}, \underset{COST}{3106.231})$

DISTANCE FROM CONNECTION BOX $\approx 7 - 2.683 \approx 4.317$ MILES

MINIMUM COST $= \$3106.23$

(60) A company must design and build a 400 ft^3 box for a product they are putting out on the market. To accommodate the size of the product, the length needs to be 4 times longer than the width. Find dimensions that will minimize the amount of carboard that is used to make it. How much cardboard is to be used to make the box?



$V = 400 \text{ FT}^3$

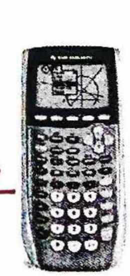
$400 = 4x^2h$

$\frac{100}{x^2} = h$

$S = \underset{L/R}{2xh} + \underset{F/B}{2\cdot 4xh} + \underset{T/B}{2\cdot 4x^2}$

$= 8x^2 + 10xh$

$= 8x^2 + 10x \cdot \frac{100}{x^2} = 8x^2 + \frac{1000}{x}$



MINIMUM $(3.969, 377.976)$
x FT^2

$x = 3.969$ FT
$4x = 15.874$ FT
$h = 6.350$ FT

MINIMUM CARDBOARD $\approx 377.976 \text{ FT}^2$

K. Willems

(61) A culture of bacteria obeys the law of uninhibited growth. If 50,000 bacteria are present initially, and there are 850,000 after 6 hours, find the exponential function $A(t)=ae^{kt}$ that models this data (round to 3 decimal places).

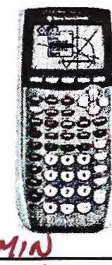
$850\,000 = 50\,000e^{6k}$

$17 = e^{6k}$

$k = \frac{\ln 17}{6} \approx 0.472$

$A = 50\,000e^{0.472t}$

(62) A cup of coffee is heated to 182˚F and is then allowed to cool in a room whose air temperature is 68˚F. After 9 minutes, the temperature of the cup of coffee is 142˚F. Find the function that represents this situation and the time needed for the coffee to cool to a temperature of 100˚F (round to 3 decimal places).



$T = T_s + (T_0 - T_s)e^{kt}$

$142 = 68 + (182-68)e^{9k}$

$k = \frac{\ln(37/57)}{9} \approx -0.048$

$T(t) = 68 + 114e^{kt}$

$100 = 68 + 114e^{kt}$

$\ln(16/57) = kt \quad t = \frac{\ln(16/57)}{k} \approx 26.460$ MIN

(63) The number of students infected with the flu at Sunnyside High School after t days is modeled by the function $F(t)=\dfrac{900}{1+59e^{-0.138t}}$.

A) What was the initial number of infected students?

$(0, 15)$ 15 STUDENTS

B) How many students will be infected after 11 days?

$(11, 64.610)$ ABOUT 65 STUDENTS

C) The school will close when 300 of the 900 student body are infected. When will school close (round to 3 decimal places)?

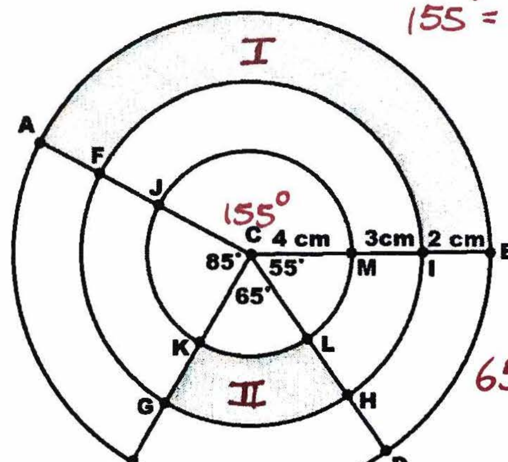
$300 = \frac{900}{1+59e^{-.138t}}$ $\quad t = \frac{\ln(2/59)}{-0.138} \approx 24.525$ DAYS

(64) Find the length of arc FI (round to 3 decimal places).

$S = r\theta$

$= 7\left(\frac{31\pi}{36}\right) = \frac{217\pi}{36} \approx 18.937$ CM

$155° = \frac{31\pi}{36}$



(65) Find the total area of the shaded regions (round to 3 decimal places).

$65° = \frac{13\pi}{36}$

$A = \frac{1}{2}r^2\theta$

I $\quad A_I = \frac{1}{2}(9)^2\left(\frac{31\pi}{36}\right) - \frac{1}{2}(7)^2\left(\frac{31\pi}{36}\right) = \frac{124\pi}{9}$

II $\quad A_{II} = \frac{1}{2}(7)^2\left(\frac{13\pi}{36}\right) - \frac{1}{2}(4)^2\left(\frac{13\pi}{36}\right) = \frac{143\pi}{24}$

TOTAL AREA $= \frac{1421\pi}{72} \approx 62.003$ CM^2

Find the sum of the series.

(66) $21+25+29+33+\ldots; n=26$ $d=4$

$S_{26}=\frac{26}{2}\left(2\cdot 21+4(26-1)\right)=1846$

(67) $2+6+18+54+\ldots; n=13$ $r=3$

$S_{13}=\frac{2(1-3^{13})}{1-3}=1,594,322$

(68) $20480+5120+1280+320+\ldots$ $r=1/4$

$S_{\infty}=\frac{20480}{1-1/4}=\frac{81920}{3}=27306\,2/3$

(69) $6+12+24+48+\ldots$ $r=2$

INFINITE GEOMETRIC SERIES DIVERGES SINCE $|r| \not< 1$

(70) $29+22+15+8+\ldots+(-160)$ $d=-7$

$-160=29-7(n-1)$

$n=28$

$S_{28}=\frac{28}{2}(29-160)=-1834$

(71) $12288+(-6144)+3072+\ldots+\left(-\frac{3}{2}\right)$ $r=-1/2$

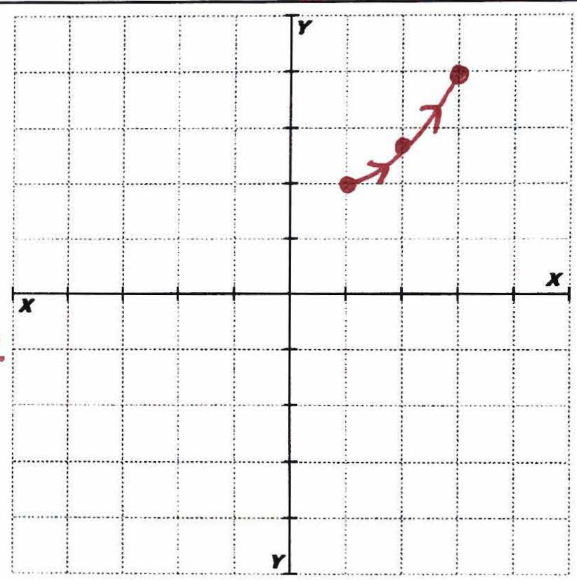
$-3/2=12288(-1/2)^{n-1}$

$n=13$

$S_{13}=\frac{12288-(-3/2)(-1/2)}{1-(-1/2)}=\frac{16383}{2}=8191.5$

(72) Sketch the parametric curve & determine if y is a function of x. Indicate the orientation of the curve (direction in which it is traced). Eliminate the parameter to find an equation that relates x & y directly.

$x=\sqrt{t-2}$ & $y=\frac{t+5}{4}$ for t in the interval $[3,11]$



t	x	y
3	1	2
6	2	11/4 = 2 3/4
11	3	4

$x^2=t-2$

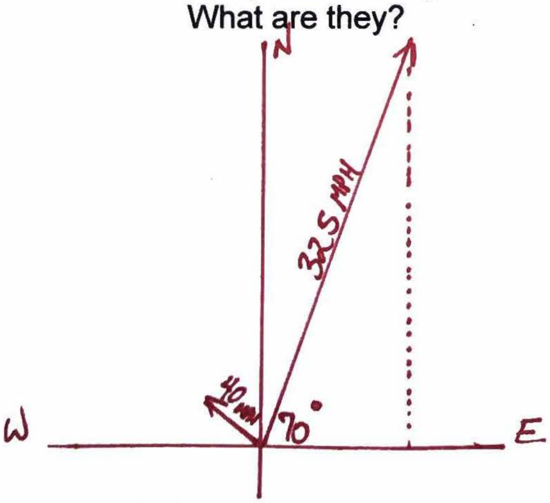
$x^2+2=t$

$y=\frac{x^2+2+5}{4}$

$=\frac{x^2+7}{4}$

$=\frac{1}{4}x^2+\frac{7}{4}$

(73) An airplane, flying in the direction 20° east of north at 325 mph in still air, encounters a 40-mph tail wind acting in the direction of 40° west of north. The airplane maintains its compass heading but, because of the wind, acquires a new ground speed and direction. What are they?



AIRPLANE $\langle 111.157, 305.400\rangle$ $\theta=70°$

WIND $\langle -25.712, 30.642\rangle$ $\theta=130°$

TOTAL $\langle 85.445, 336.042\rangle$

MAGNITUDE = SPEED ≈ 346.735 MPH

$\tan\theta=\frac{y}{x} \longrightarrow \theta \approx 75.734°$ NORTH OF EAST